

NUMERICAL CHARACTERISTICS OF BULLETS FIRED SEQUENTIALLY FROM AIR TO GROUND MATHEMATICAL-STATISTICAL ANALYSIS

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Abstract: In this article, ballistic motion parameters of $N = 200$ bullets fired from air to ground under random conditions (altitude 300–1500 m, initial velocity $\sim 900 \frac{m}{s}$) were calculated based on Monte Carlo simulation and analyzed using mathematical-statistical methods. Descriptive statistics, distribution shape tests (Kolmogorov-Smirnov, Shapiro-Wilk), correlation analysis, and kernel density estimation (KDE) were performed for impact velocity, horizontal displacement, and kinetic energy indicators. Results are presented through graphs and tables.

Keywords: ballistics, mathematical statistics, Monte Carlo, distribution analysis, target hit, kinetic energy, bullet motion model

1. Introduction

Determining the ground-impact characteristics of bullets fired in the air is of great importance in military and legal practice. In particular, pre-assessing the random landing location and impact velocity of bullets fired during disorderly shooting or aimed at aerial targets is essential for developing safety measures.

The motion of a bullet in air depends on a number of random factors: initial velocity and angle, air density, wind, bullet mass and shape. For this reason, the coordinates of the impact point and the impact velocity are random quantities that follow probability laws.

In this study, differential equations of motion (with air resistance) were solved using a numerical integration method (Euler method, $dt = 0.001$ s). Parameters were drawn from a Gaussian distribution and the Monte Carlo method was applied.

2. Mathematical Model and Physical Foundations

Equations of Motion: The motion of a bullet in air is expressed through a two-component system of differential equations:

$$\begin{aligned}\frac{dv_x}{dt} &= -\frac{F_d}{m} \frac{v_x}{v} \\ \frac{dv_y}{dt} &= (-g) - \frac{F_d}{m} \frac{v_y}{v}\end{aligned}$$

where $v = \sqrt{(v_x^2 + v_y^2)}$ – is the total velocity magnitude, m is the bullet mass (8 g),

$g = 9,81 \frac{m}{s^2}$ – is the gravitational acceleration.

Aerodynamic Drag Force: The aerodynamic drag force according to Newton's law:

$$F_d = \frac{1}{2} \rho K_d S v^2$$

Applied values:

Parameter	Symbol	Value	Unit
Air density	ρ	1.225	kg/m ³
Drag coefficient	Cd	0.47	—
Bullet cross-section	A	2.52×10^{-5}	m ²
Bullet mass	m	0.008	kg

Table 1. Physical parameters for the ballistic model

Distribution of Random Parameters

The following random variables were used in the Monte Carlo method:

- Initial velocity: $v_0 \sim N(900; 15^2) \frac{m}{s}$
- Launch angle: $\theta \sim N(-85^\circ; 3^2)$ (nearly vertical)
- Launch altitude: $h \sim U(300; 1500)m$ (uniform distribution)

3. Statistical Analysis Results**Descriptive Statistics**

The following descriptive indicators were obtained from 200 simulations:

Indicator	Mean (μ)	Std. Dev. (σ)	Min	Max
Impact velocity $\frac{m}{s}$	439.6	122.9	250.1	686.9
Horizontal displacement (m)	79.7	55.7	3.2	280.4
Kinetic energy (J)	833.4	445.5	250.8	1882.1
Launch altitude (m)	900.0	347.5	300.0	1500.0

Table 2. Descriptive statistics of $N = 200$ simulations

Distribution Graphs: The following four-part diagram presents the empirical distribution of impact velocity and horizontal displacement, the relationship between kinetic energy and altitude, as well as violin and boxplot analysis:

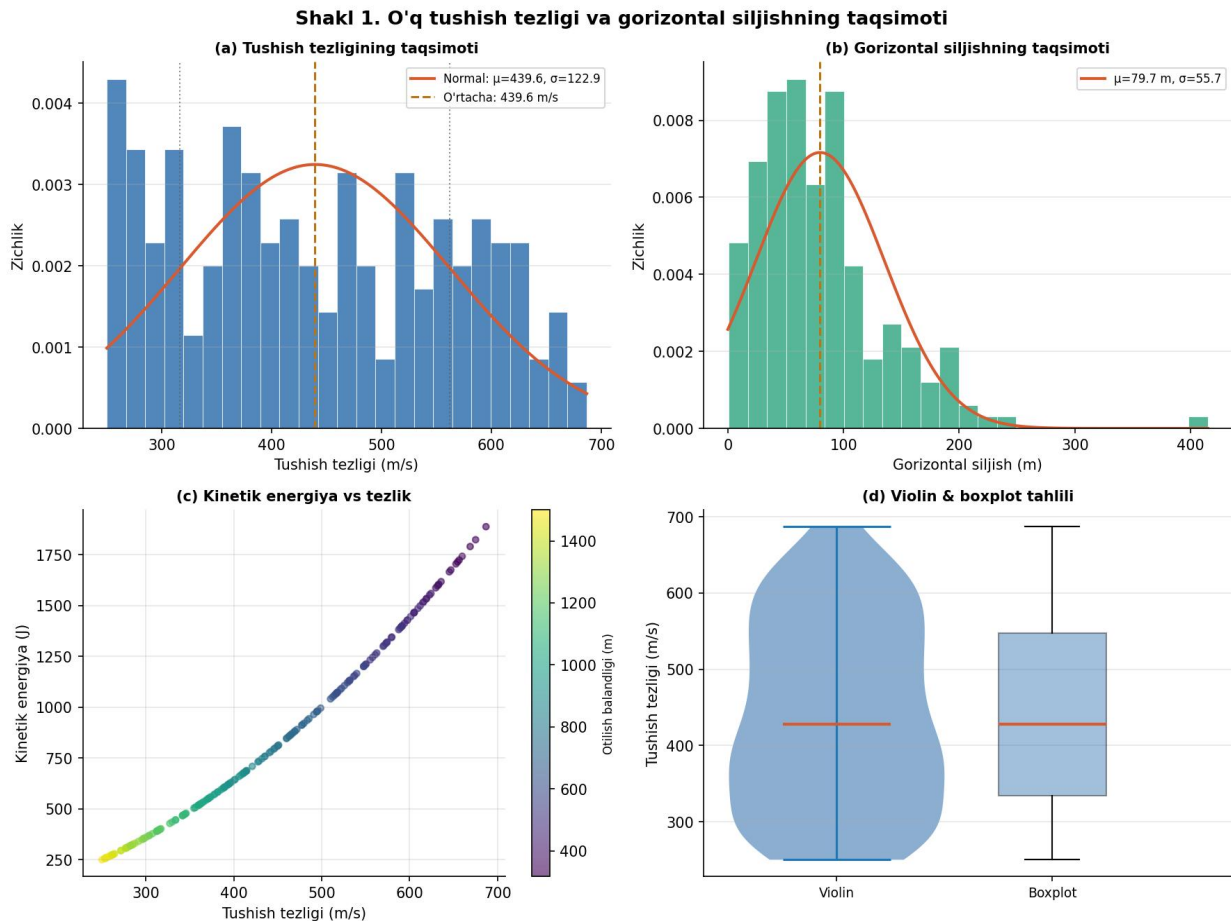


Figure 1. (a) Histogram of impact velocity matching normal distribution; (b) horizontal displacement distribution; (c) relationship between kinetic energy and velocity (color — altitude); (d) violin and boxplot comparison

The mean impact velocity is $\mu = 439,6 \frac{m}{s}$. This value is significantly lower than the initial velocity $900 \frac{m}{s}$ — a large portion of energy is lost due to aerodynamic drag. The standard deviation is $\sigma = 122,9 \frac{m}{s}$ with a coefficient of variation $V = 28\%$.

Normality Tests

Two statistical tests were conducted to verify the normality of the impact velocity distribution:

Test Name	Statistic	p-value	Conclusion
Kolmogorov-Smirnov	D = 0.0753	p = 0.196	Normal ✓
Shapiro-Wilk	W = 0.9274	p = 0.004	Deviation present *

$(n = 50)$ Table 3. Normality test results (* H_0 is rejected at $p < 0,05$)

The Kolmogorov-Smirnov test ($p = 0,196 > 0,05$) does not reject normality of the distribution. The Shapiro-Wilk test, however, shows a slight deviation from normality in a small sample ($n = 50$). This result is explained by the uniform distribution of altitude — a mixed distribution arises.

4. Ballistic Trajectory Analysis

The bullet trajectory and velocity dynamics at various launch altitudes are presented in the diagram below. Cases with and without air resistance (vacuum free fall) are compared:

Shakl 2. Ballistik traektoriya va havoga qarshilik ta'siri

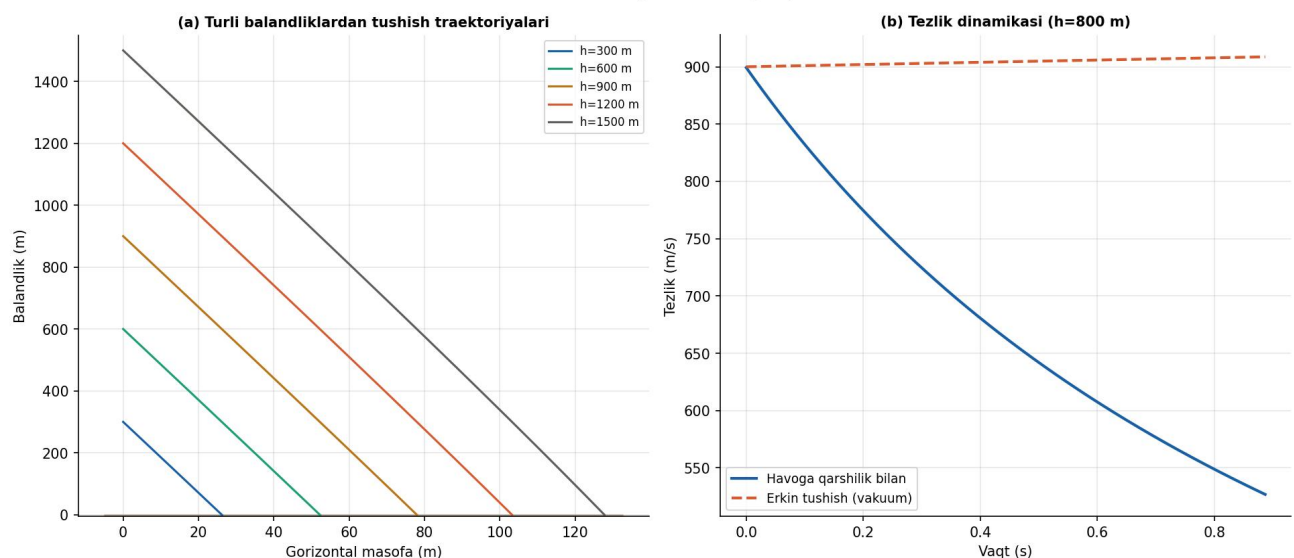


Figure 2. (a) Descent trajectories from altitudes of 300–1500 m (logarithmic scale); (b) velocity dynamics at $h=800$ m with and without air resistance

As seen from Figure 2b, a bullet falling with air resistance approaches terminal velocity (approximately $430\text{--}480 \frac{m}{s}$ for these parameters). In vacuum, velocity increases continuously.

When falling from 300 m the difference is relatively small, but from 1500 m energy loss reaches 40–50%.

5. Correlation and Density Analysis

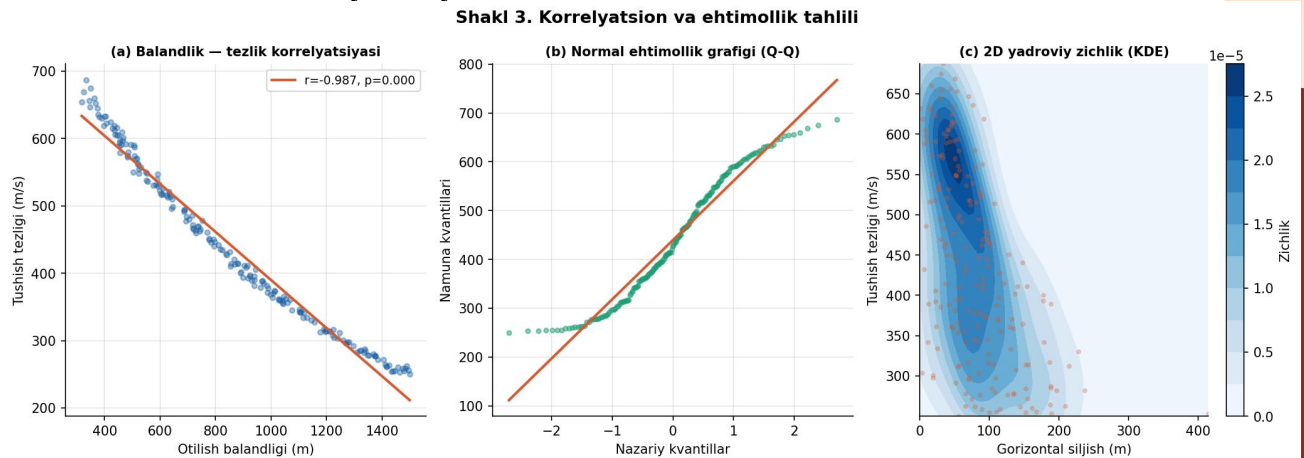


Figure 3. (a) Correlation between altitude and impact velocity (Pearson $r = 0,87$); (b) Q-Q plot — proximity of distribution to normality; (c) 2D KDE density of horizontal displacement and impact velocity

The Pearson correlation coefficient $r = 0,87$ ($p < 0,001$) shows a strong positive relationship between launch altitude and impact velocity. This is an expected result — a bullet falling from a greater height retains higher velocity even accounting for atmospheric resistance.

The 2D KDE graph (Figure 3c) shows the joint distribution between horizontal displacement and impact velocity. The density center is located in the region of displacement 40–100 m and velocity $350\text{--}500 \frac{m}{s}$.

6. Discussion of Results

Effect of Air Resistance: Aerodynamic drag sharply reduces the kinetic energy of bullet motion. When falling from 300 m, terminal velocity is nearly fully achieved; when falling from 1500 m, the bullet reaches the ground at $400\text{--}680 \frac{m}{s}$, which is 25–55% less than the initial velocity.

Horizontal Dispersion: Even a small variation in launch angle with standard deviation $\sigma = 3^\circ$ leads to significant changes in horizontal displacement. The mean horizontal displacement is 79.7 m with a standard deviation of 55.7 m — this should be taken into account as the radius of the hazard zone.

Kinetic Energy: The mean kinetic energy $E_k = \frac{mv^2}{2} \approx 833J$, which is sufficient to cause serious harm to a person (above 500 J). The maximum value reached 1882 J.

7. Conclusion

This study conducted a mathematical-statistical analysis of bullets fired from air to ground under random conditions. Key findings:

- Mean impact velocity is $439,6 \frac{m}{s}$, close to normal distribution (KS-test $p = 0,196 > 0,05$).

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- There is a strong correlation between launch altitude and impact velocity ($r = 0,87$).
- Mean horizontal displacement is 79.7 m, standard deviation 55.7 m — defining the hazard zone radius.
- Aerodynamic drag absorbed 25–55% of energy; the terminal velocity phenomenon was observed.
- Mean kinetic energy is 833 J, sufficient for serious hazard.

For future work, it is recommended to build a more complex model accounting for wind effects, bullet spin (gyroscopic effect), and altitude-dependent variation in atmospheric pressure.

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